An experimental investigation of heat transfer effects on boundary layer separation in supersonic flow

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SUMMARY

Experiments have been done on the effects of heat transfer on wall-pressure distributions through separated regions with both laminar and turbulent boundary layers at a free-stream Mach number of about 3. The temperature of the flat plate on which the boundary layer was formed could be varied from about -35° C to $+75^\circ$ C. According to theory, this variation should have produced appreciable alterations at a laminar separation point in either the pressure or the pressure gradient, but no sign of this appeared in the overall pressure distributions, which, for laminar layers, remained unaffected by wall temperature. A possible explanation is given for this apparent discrepancy between theory and experiment. With turbulent layers, the variations in wall temperature did produce small changes in the pressure distributions. However, for most practical purposes such changes could be ignored. Hence the convenient conclusion is suggested that in supersonic separating flow with either a laminar or a turbulent boundary layer the pressure distributions are not significantly affected by moderate variations in wall temperature.

1. INTRODUCTION

Several theoretical papers (Cohen & Reshotko 1955a, 1955b; Gadd 1952, 1956a; Illingworth 1954; Morduchow & Grape 1955; see also Gadd 1956b) have been written on the problem of the effects on laminar separation of cooling or heating the wall. (By cooling is meant maintaining the temperature T_w of the wall at less than the value T_{wz} for which there is zero heat transfer between fluid and wall. T_{wz} is, of course, usually a little below the stagnation temperature.) All the theories agree in predicting that cooling the wall makes it more difficult to separate a laminar boundary layer, and that heating has the opposite effect. No theory has yet been developed for the effects of heat transfer on turbulent separation, but one would expect there to be effects qualitatively similar to those on laminar layers. However the effects ought to be of smaller magnitude, one would think, since, in the boundary layer upstream of the region of separation, changes in wall temperature produce smaller proportional changes of velocity and density when the flow is turbulent than they do when it is laminar. With laminar layers, for an arbitrarily fixed pressure distribution, it is typically found in the theories that the pressure coefficient* C_{p_s} at the separation point roughly obeys the relation $C_{p_s} \propto (T_w/T_{wz})^{-n}$, where n is between 0·5 and 1, and T_w is measured on the absolute scale. This considerable predicted effect of heat transfer is of great interest, because in many practical applications where boundary layer separation at supersonic speeds may occur the wall temperature will be much lower than the zero heat transfer value. Also, in certain wind tunnel investigations observations

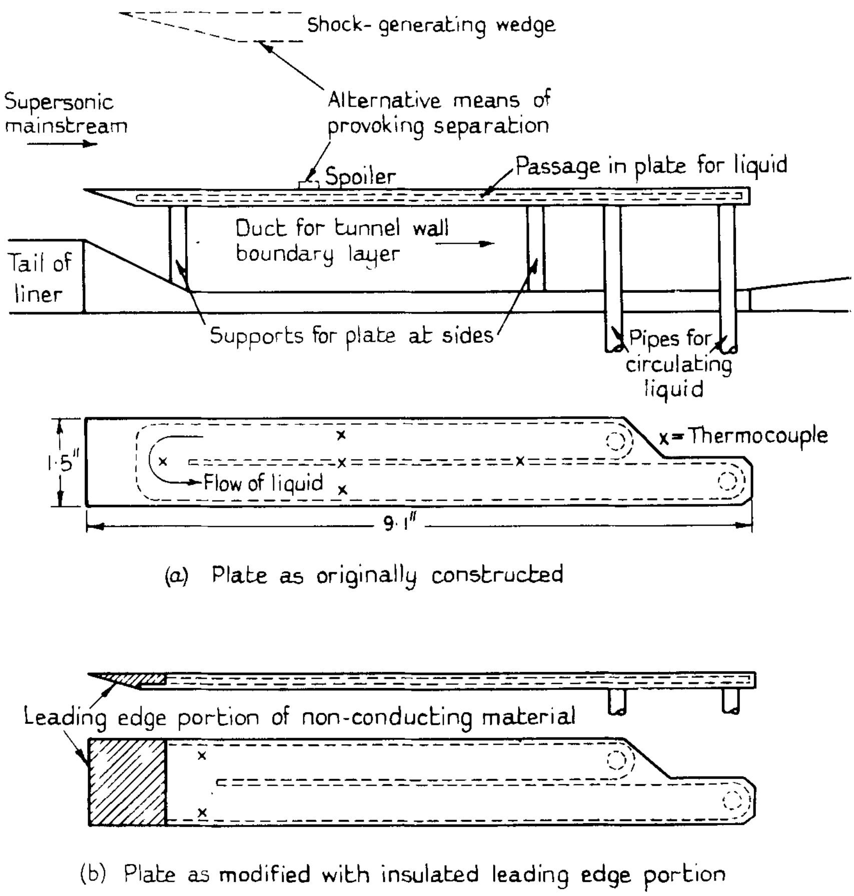

Figure 1. The flat plate on which the test boundary layer was formed.

are made before the model and flow are in thermal equilibrium, so that errors may arise if the effects of heat transfer on separation are appreciable. Previous experimental work (for example Gadd, Holder & Regan 1954) on separation in supersonic flow has all been concerned with the insulated condition. Hence it was decided to investigate experimentally cases of laminar and turbulent separation with the wall heated or cooled.

* The pressure coefficient $C_p = 2(p-p_1)/\gamma M_1^2 p_1$ where p = pressure, p_1 = free-stream pressure, γ = ratio of specific heats, and M_1 = free-stream Mach number.

The experimental arrangement was as shown in figures 1 and 2. The experiments were done at a Mach number of about 3. The test boundary layer was formed on a hollow flat plate through which hot or cold liquid could be circulated. Separation could be provoked either by a shock wave generated by a wedge held in the mainstream above the plate, or by a spoiler attached to the surface. With either arrangement separation occurs upstream of the agency provoking it. The pressure distribution in the neighbourhood of separation is thus governed by the equilibrium between the thickening of the boundary layer and the associated deflection of the external flow from its original free-stream direction. Most of the theories for laminar layers mentioned above do not take account of this equilibrium process which will nearly always in practice govern the pressure distribution.

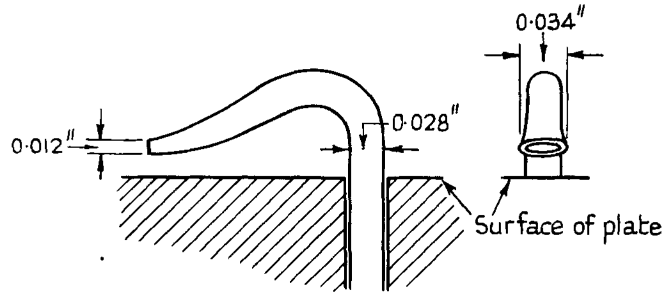

Figure 2. Pitot tube for boundary layer traverses.

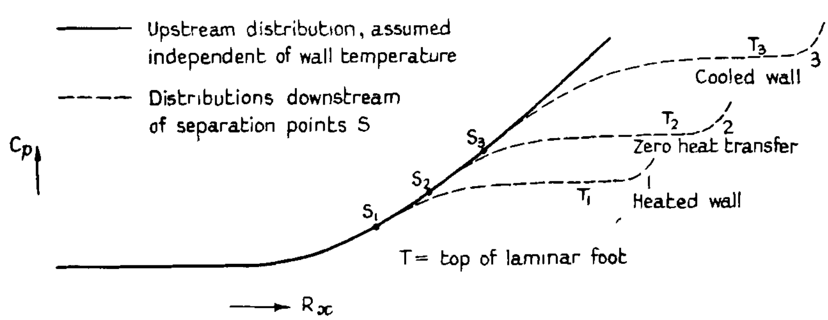

Figure 3. Possible distributions of the pressure coefficient C_p for laminar separations as a function of the Reynolds number R_x based on free-stream conditions and the distance x from the leading edge.

However, one would expect the theoretical results for cases with arbitrary pressure distributions to have a qualitative relevance to the experimental laminar case. They would certainly be relevant if it so happened that, for a given free-stream Mach number and a given Reynolds number at the upstream end of the compression region, the pressure distribution as far as the separation point always formed part of a single curve, independent of wall temperature, as shown in figure 3. Then, according to the theories, the pressure coefficient C_{p_s} at the separation point would increase as the wall temperature was reduced. It would seem natural in these circumstances for the pressure distribution downstream of separation to be of the shape shown in figure 3, and to vary with wall temperature as shown there, for the following reasons. When laminar separation occurs in practice, the pressure gradients downstream of separation fall off where the dead-air region becomes thick, and usually increase again further downstream because of transition to turbulent flow. Thus the pressure distribution shows a 'laminar foot' (Gadd, Holder & Regan 1954), with a fairly well defined 'top' at which the pressure coefficient C_{p_T} may be readily determined. With zero heat transfer C_{p_T} is roughly proportional to C_{p_s}, and at first sight there seems no reason why the ratio of C_{p_T} to C_{p_s} should vary greatly with wall temperature. Thus, for a fixed upstream pressure distribution as shown in figure 3, C_{p_T}, as well as C_{p_s}, would be expected to vary like T_w^{-n}, where n is between 0·5 and 1. However, more will be said on this point later. Meanwhile it must be borne in mind that it is far from evident that the pressure distribution upstream of the separation point ought in reality

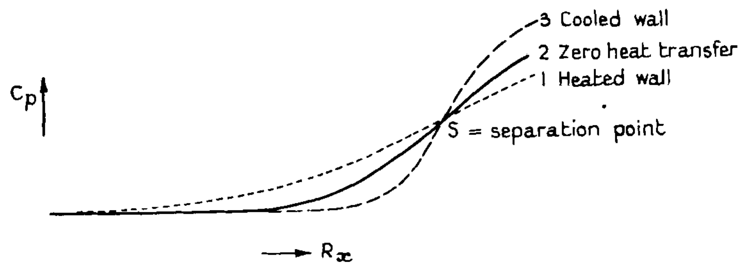

Figure 4. The variation with wall temperature of the pressure-coefficient distribution with laminar separation, according to Gadd (1956 a).

to be independent of wall temperature, as shown in figure 3. According to Gadd (1956a) the pressure distribution varies with wall temperature not only downstream of the separation point, but upstream of it as well. Gadd's theory is based on more realistic assumptions than the other theories mentioned inasmuch as it takes account of the equilibrium between the pressure gradients and the thickening of the boundary layer. It gives the result, which might appear to contradict the other theories for laminar layers, that the pressure coefficient at separation is unaffected by wall temperature, as shown in figure 4. However, a further result is that the pressure gradients at separation are sharper when the wall is cooled and more gradual when it is heated, since they vary as $T_w^{-3/2}$. If, with a given constant wall temperature, the pressure distribution could be changed from the form (1) of figure 4 to the form (3), without regard to the thickening of the boundary layer, separation would occur at a lower pressure with the form (3). Thus the theoretical result (Gadd 1956a) that separation occurs at the same pressure with cooling despite sharper gradients is, in effect, consistent with the idea that cooling makes it more difficult to separate the boundary layer.

However, despite the qualitative agreement of all the theories, the experimental results for laminar layers appeared to contradict them. The surprising result was obtained that the maximum pressure gradient at the upstream end of the laminar foot, the pressure coefficient C_{p_T} at the top of the foot, and indeed the entire wall-pressure distribution, are all virtually unaffected by cooling or heating the wall over the range -35° C to $+75^\circ$ C. The wall temperature for zero heat transfer is always near 0° C, a little below the stagnation temperature (which is approximately atmospheric temperature). Hence the maximum cooling employed represents a proportional decrease of about $\frac{1}{8}$ in absolute temperature, and the heating an increase of about $\frac{1}{4}$, so that the theories would certainly lead one to expect observable effects. Perhaps equally surprising is the result that there are bigger observable effects with turbulent boundary layers than with laminar ones. However, it may well be that there are real effects with laminar layers which are not apparent in the overall pressure distributions. Further discussion of the interpretation of the results is given in a later section.

2. Description of apparatus

Figure 1 is a simplified diagram of the flat plate on which the test boundary layer was formed. The plate completely spanned the 2·6 in. × 1·5 in. blow-down tunnel (Gadd, Holder & Regan 1954) in which the investigations were made. Along the centre line of .the plate there were pressure tappings and there were also thermocouples distributed over the surface as shown. The thermocouple heads were soldered into $\frac{3}{16}$ in.-diameter brass plugs which were forced into tightly fitting holes in the plate, and made flush with the surface. At the end of the investigations the plate was modified as in figure 1 (*b*) to have an insulated portion at the front.

The liquid that was circulated through the $\frac{1}{10}$ in.-deep passage in the plate was made to flow at about 5 to 10 ft./sec. Before entering the plate, it was pumped through a coil of copper pipe which served as a heat exchanger. The coil was placed in a large thermos flask so that it could be surrounded by liquid. When it was desired to cool the plate, methylated spirit was used for the circulating liquid and the liquid in the flask. Into the latter, lumps of solid carbon dioxide were dropped, and by this method the plate could be cooled to about -35° C when low stagnation pressures were used for investigations on laminar layers near the front of the plate. For investigations on turbulent layers further along the plate, higher stagnation pressures were used to give the requisite higher Reynolds numbers. This meant that the heat transfer was greater, and under these conditions the minimum temperature attainable was about -25° C. For heating the plate water was used as the circulating liquid and was heated by nearly-boiling water which was put in the flask. Plate temperatures of about 75° C for laminar layers and 65° C for turbulent ones were attainable by this means. Heat conduction ensured that all metal parts of the plate surface (except perhaps those very close to the leading edge) took up a temperature which was roughly uniform.

Owing to the space required by the circulating liquid, the plate had to be thicker than the one used in previous investigations of interactions between shock waves and boundary layers (Gadd, Holder & Regan 1954). This increased thickness and the pipes ducting the liquid in and out of the plate made the blockage rather large, and at low stagnation pressures, like those used in the investigations of laminar flow, prevented the use of a shock-generating wedge of large angle. Thus large regions of laminar separated flow were not readily obtainable by means of a wedge in the free stream, and the technique used in the former experiments of moving the wedge axially in order to move the pressure distribution past the pressure tappings was not entirely satisfactory. This limitation was finally overcome by drilling extra pressure tappings. The original spacing had been $\frac{1}{2}$ in., and the later spacing was $\frac{1}{6}$ in. in the region used for investigations of laminar flow and, finally, $\frac{1}{12}$ in. in the partly-insulated version of the plate shown in figure 1 (*b*). The more closely spaced holes made it possible to obtain satisfactory pressure distributions with laminar separations by using a fixed spoiler about 0·04 in. high glued to the surface at right angles to the flow and completely spanning the plate and tunnel. This arrangement provoked separation about $\frac{1}{2}$ in. to $\frac{3}{4}$ in. upstream of the spoiler.

A thin Pitot tube of external diameter 0·028 in. could be arranged to protrude through either of two of the static pressure tappings, one in the region where the laminar separations were made to occur, and the other in the region of the turbulent separations. The tube was bent over at the top and flattened at the tip to a height of 0·012 in. and a width of 0·034 in., as shown in figure 2. A micrometer underneath the working section moved it up and down so that pitot traverses of the boundary layer could be made.

3. Results and discussion

(*a*) *Laminar separations*

Figures 5 to 9 show pressure distributions for laminar separations. The pressure p at the surface, made non-dimensional by dividing it by p_1, the constant value of p in the undisturbed region upstream of separation, is plotted against the distance x along the plate from the leading edge. The Reynolds numbers R_x based on x and free-stream conditions are marked on the abscissa scales. It can be seen that there is no appreciable systematic effect of temperature in any of the figures, and mean curves have accordingly been drawn through the experimental points. The curves all show the characteristic laminar foot, with moderate pressure gradients at the upstream end, a flatter top to the foot, and a steep increase of pressure at the downstream end. This latter increase is due to transition to turbulent flow, which in the present experiments always occurred within the separated flow region when the flow was laminar at separation.

Figures 5 to 7 were obtained with the all-metal plate as originally constructed, i.e. they correspond to an approximately uniform wall temperature. With the plate modified as shown in figure 1 (*b*) to have an insulated portion near the leading edge, the wall temperature over the insulated portion always assumes the zero heat transfer value, near to 0° C. Hence when hot or cold liquid is circulated there is an abrupt step of wall temperature at the junction with the metal portion. Figures 8 and 9 correspond to this condition.

Figures 6 and 7 differ from the rest in certain respects. The different figures correspond to distributions obtained on different occasions, the flat plate having been removed from the tunnel and put back again in between times. In figure 6 the laminar foot is rather short, the separated layer being more ready to turn turbulent than on other occasions. This variation in transition behaviour may be due to a variation in the condition of the leading edge. An alternative possible cause is a variation in the

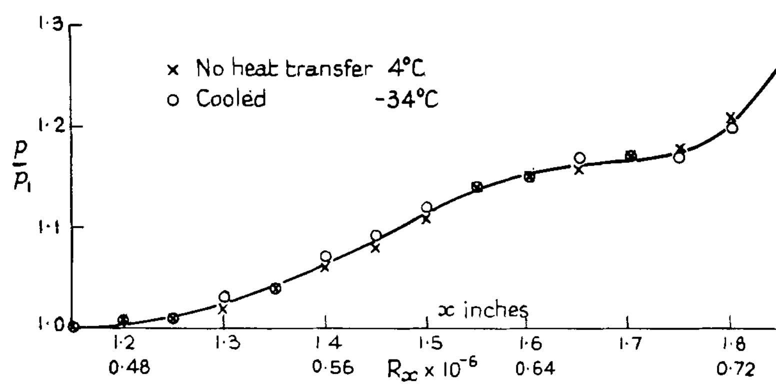

Figure 5. Pressure distributions with laminar separation induced by a wedge in the mainstream. Stagnation pressure 2 atmospheres.

disturbance due to air leaking round the sides of the plate from underneath near the leading edge, although an attempt was always made to seal off this leakage. In figure 7 the non-dimensional pressure gradient

$$x\frac{d}{dx}\left(\frac{p}{p_1}\right)$$

over the upstream part of the laminar foot is much greater than in the other figures. This is probably due to disturbances emanating from the leading edge at the corners where it meets the side walls of the tunnel. Such disturbances would cross the plate centre line, on which the pressure tappings are situated, at about 2·1 in. from the leading edge at a Mach number of 3, and could well be responsible for distorting the pressure distributions a little upstream of this position. The other distributions are nearer the leading edge, and so are not affected. However, the distortions in figure 7, and the effects of the early transition in figure 6, should be independent of temperature, so that the two figures are admissible as evidence that temperature has little effect on the pressure distributions.

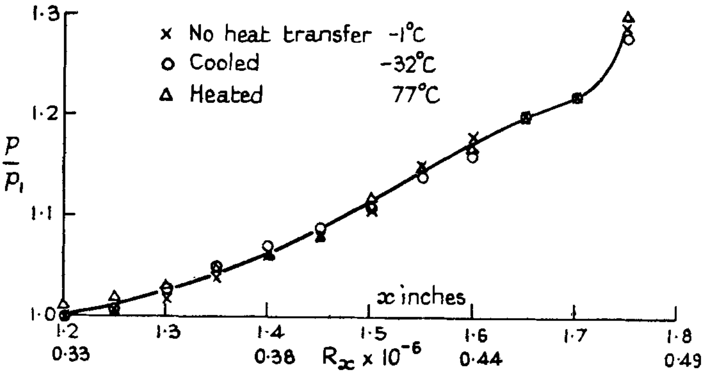

Figure 6. Pressure distributions with laminar separation induced by a wedge in the mainstream. Stagnation pressure 1·3 atmospheres.

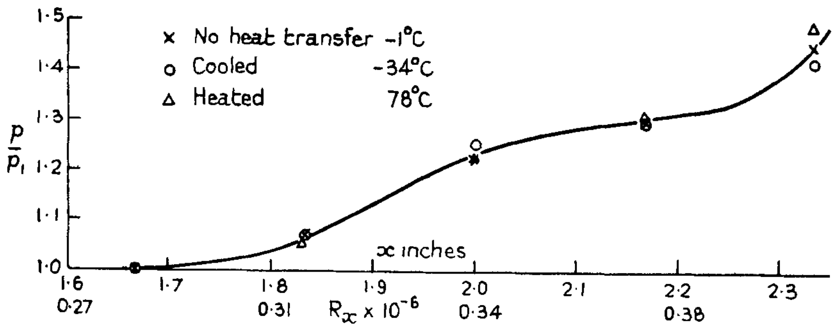

Figure 7. Pressure distributions with laminar separation induced by a spoiler. Stagnation pressure 0·8 atmospheres.

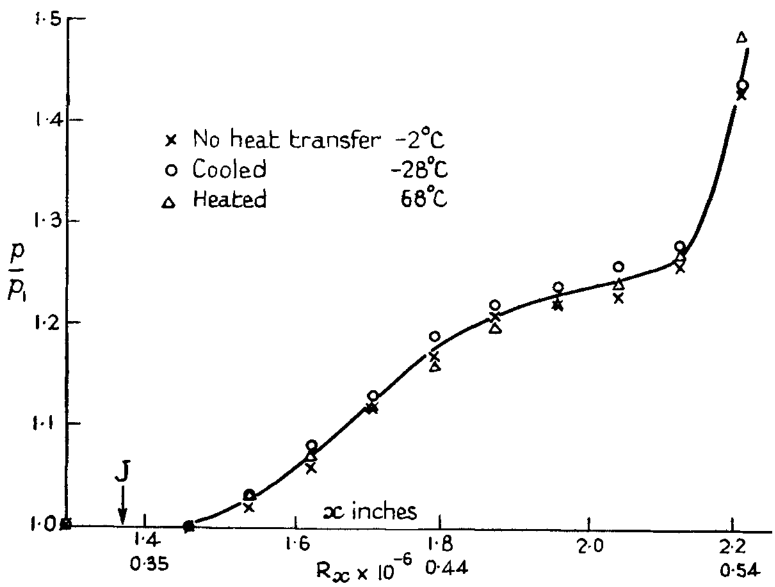

Figure 8. Pressure distributions with laminar separation induced by a spoiler. *J* marks the junction between the insulated leading edge portion and the metal. Stagnation pressure 1·2 atmospheres.

Nevertheless, temperature does affect the boundary layer flow, as can be seen from figure 10. This shows Mach number profiles measured in the undisturbed boundary layer about 1·2 in. from the leading edge with the separation-provoking agency (wedge or spoiler) removed. The Reynolds number R_x was $0{\cdot}34 \times 10^6$ at the measurement position. The Mach number was deduced from the static pressure at the wall and the readings of the pitot tube traversed through the boundary layer. In figure 10 the Mach numbers M are plotted against the distances z_T from the wall to the centre of the Pitot tube opening. Traverses made on different occasions did not repeat exactly, perhaps because of certain imperfections in the micrometer mechanism. For this reason the Mach number profiles for the different wall temperatures are first plotted separately showing the experimental

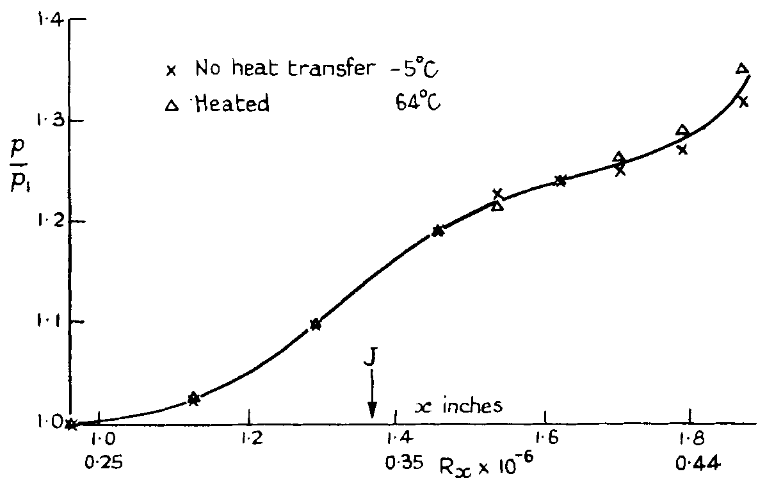

Figure 9. Pressure distributions with laminar separation induced by a spoiler. J marks the junction between the insulated leading edge portion and the metal. Stagnation pressure 1·2 atmospheres.

points, in figures 10(*a*), (*b*), and (*c*). The different symbols for the points correspond to traverses made on different occasions. The mean curves are superimposed in figure 10(*d*). Despite the scatter of the experimental points it is clear that there are real differences in the boundary layer profiles. The experimental points nearest the wall in figures 10(*a*), (*b*), and (*c*), lie above the mean curves, especially in the zero heat transfer and heated cases. These points are obtained with the Pitot tube resting on the surface. The reading then corresponds to the Pitot pressure at a point further away from the wall than the centre of the tube opening. Taylor (1938) showed that this displacement of the effective position of measurement decreases as the Reynolds number

$$R_d = d^2\left(\frac{\rho}{\mu}\frac{\partial u}{\partial z}\right)_{\text{wall}}$$

increases, where d is the height of the Pitot at the tip, z is distance from the wall along the normal, and the other symbols have their usual significance.

For a given stagnation pressure and a given distance from the leading edge, the skin friction should be little affected by wall temperature (Howarth 1953, pp. 418 to 425). The viscosity μ is approximately proportional to absolute temperature, whilst the density ρ is inversely proportional to it, so that $R_d \propto T_w^{-3}$ approximately, where T_w is the absolute temperature at the wall. Hence, according to Taylor (1938), the effective displacement should be greatest with the heated wall, as is found experimentally. The theoretical and experimental values for the magnitude of the displacement also agree reasonably.

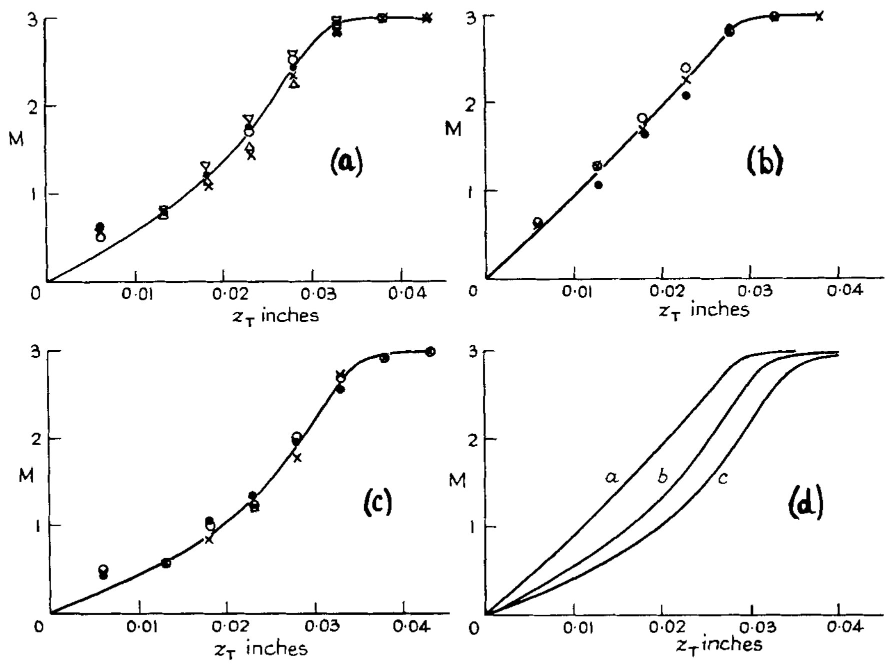

Figure 10. Laminar Mach number profiles ; (*a*) with zero heat transfer (stagnation pressure 1·3 atmospheres, wall temperature about 6° C) ; (*b*) with a cooled wall (stagnation pressure 1·3 atmospheres, wall temperature about −34° C) ; (*c*) with a heated wall (stagnation pressure 1·3 atmospheres, wall temperature about 73° C) ; (*d*) the three curves superimposed (*a* cooled well, *b* zero heat transfer, *c* heated wall).

The measured boundary layer thicknesses are about twice as great as the theoretical values for a flat plate boundary layer (Howarth 1953, p. 406). However, it appears from schlieren photographs that the boundary layer is probably not really as thick as the measurements indicate. The pitot tube used is rather large, with a tip height of about $\frac{1}{3}$ the measured boundary layer thickness. Moreover, the tip is inclined downwards to the plate, as shown in figure 2, to ensure that it can make contact with the wall. Hence, since the tube is partially flattened at the tip so that its width is about three times its height, it must present a considerable disturbance to the flow, and it is quite likely that it causes a local thickening of the boundary layer. Alternatively, the discrepancy between theory and the measured profiles may possibly be due to leading edge conditions. The under surface of the plate near the leading edge (see figure 1) is inclined at about 18° to the upper surface, so that the pressures underneath must be much greater than those on the upper surface. Hence, since the leading edge shock must be slightly detached because of the inevitable slight bluntness of the edge and the presence of a boundary layer, it is possible that air leaks round from underneath at the leading edge, causing a small bubble of separation on the upper surface. This would make the boundary layer thicker than the theory predicts (cf. Bradfield, de Coursin & Blumer 1954).

The qualitative effects of heat transfer on the measured boundary layer profile shapes are similar to the theoretical ones, although the profile with zero heat transfer is closer to the heated wall profile than to that for the cooled wall, despite the fact that the temperature increase with heating is bigger than the drop with cooling. This anomalous result is probably due to inaccuracies in the profile measurements. However, even if all the discrepancies between the theoretical and measured profiles are real, and due to some unknown experimental imperfections in the flow, it hardly seems likely that these imperfections can be such as to annul exactly all the theoretically predicted effects of heat transfer on separation. Theory, as pointed out in § 1, indicates that cooling the wall should in effect make the boundary layer more difficult to separate. According to Gadd (1956 a) the maximum pressure gradient at the upstream end of the laminar foot should be increased by cooling and reduced by heating in the ratio $T_w^{-3/2}$. Thus the pressure gradient for the cooled case should be about 1·8 times as great as that for the heated case with the temperatures of the present experiments. Such a large variation in gradient should be easily detectable experimentally, so that the present results definitely disprove Gadd's predictions. However, his theory may in a sense be qualitatively correct inasmuch as, like the other theories, it predicts that cooling makes it more difficult, in effect, to separate the boundary layer. Indeed it seems inconceivable that all the theories could be wrong on this point. Hence it seems that the position of separation must occur at a higher pressure when the wall is cooled. This can still be the case even though the pressure distribution through the separated region remains substantially unaltered. The precise position of the separation point cannot be determined merely by inspection of the overall pressure distribution, though it is known to be situated somewhere on the relatively steep upstream part of the laminar foot. It is possible, therefore, that the situation is as shown in figure 11, with separation occurring further upstream on the foot at the higher temperatures. In principle this could of course be verified experimentally using a surface pitot technique (Gadd, Holder & Regan 1954), but it is difficult to get accurate results by this method on the small scale of the present experiments. Hence it was not attempted with the present apparatus because considerable modifications would have been necessary and the blockage difficulties mentioned in § 2 above would have made the technique even more difficult than usual. It is hoped to make further investigations when a larger wind tunnel becomes available.

At first sight it would seem natural for the pressures at the top of the laminar foot to be lower as in figure 3 if the pressure at separation was lower. If figure 11 represents the true state of affairs, the pressure gradients must fall off relatively more rapidly downstream of separation when the wall is cooled than they do when it is heated, and this requires explanation. A possible reason is as follows.

When the wall is uniformly cooled the velocity profile of the upstream boundary layer has a 'fuller' shape than when the wall is insulated (cf. figure 10(d)). In the case of a plate as in figure 1(b) with an insulated leading edge portion and a metal portion downstream which is cooled, the low-velocity part of the boundary layer rapidly assumes the fuller form on reaching the cooled part of the surface. This is because just downstream of the temperature discontinuity the viscosity at the wall is less than that in the middle of the boundary layer. Hence if the boundary layer profile remained the same as upstream there would be an unbalance of forces on

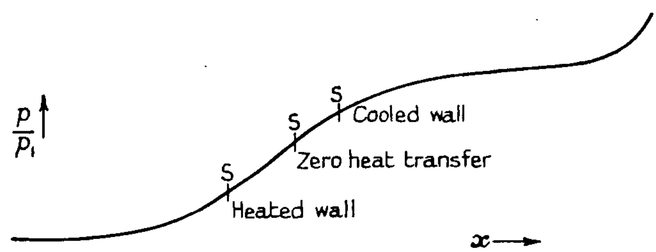

Figure 11. Suggested way in which heat transfer may in practice affect the position of the laminar separation point S.

the fluid nearest the wall—the frictional force exerted by the wall tending to retard the inner layers of fluid would be outweighed by the force tending to drag them on exerted by the fluid further out. This physical argument is confirmed by a mathematical solution which can be obtained for the flow near the wall near a temperature discontinuity. According to this solution the skin friction remains constant across the discontinuity if the viscosity is proportional to absolute temperature. Hence as the flow passes the discontinuity, the velocity gradient at the wall immediately assumes the value that it would have if the wall temperature over the insulated upstream portion were the same as that of the cooled part downstream. A little way from the wall immediately downstream of the discontinuity the velocity gradient remains approximately the same as if the wall were still insulated, so that the profile near the wall becomes curved, as shown in figure 12. However, the thickness of this region of curvature increases rapidly, varying as $(x-x_J)^{1/3}$, where $(x-x_J)$ is the distance downstream of the discontinuity. Hence the low-velocity part of the boundary layer profile very soon approaches the form that it would have if the wall were uniformly cooled.

This fuller upstream profile tends to make the boundary layer less easy to separate since the fluid fairly near the wall moves faster and is therefore less easily brought to rest. This effect is reinforced by the fact that the density is increased by cooling, so that the momentum of the fluid is still further increased. On the other hand the viscosity near the wall is decreased by cooling. This is a factor making for easier separation, because it is only viscosity that prevents the slowest moving fluid nearest the wall from being brought to rest immediately by the slightest pressure increase. However, the theories all agree that the combined effect of the fuller upstream profile and the greater density should outweigh the effect of the reduced viscosity,

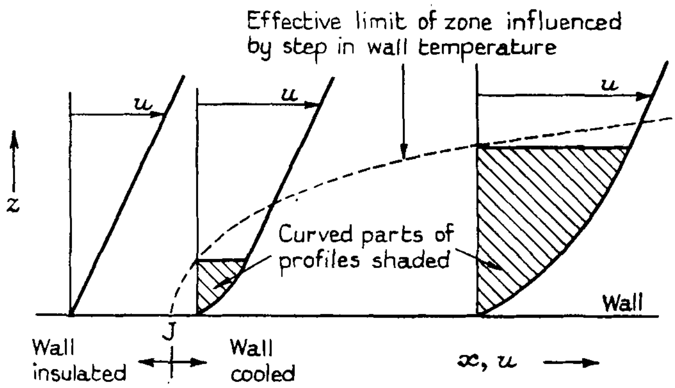

Figure 12. Laminar velocity profiles near the wall near the juntion J between the insulated and cooled portions of the wall. z = distance normal to the wall, u = velocity.

and make the boundary layer more difficult to separate. Once separation has occurred, however, it seems quite possible that the viscosity effect may become dominant. The region downstream of separation is, as it were, further removed from the influence of the upstream profile. Hence it may quite well be that the pressure gradient which the boundary layer is able to withstand falls off more rapidly downstream of separation when the wall is cooled. This could account for the fact that the overall pressure distribution remains unaltered, as in figure 11, despite the assumed increased pressure at separation.

The apparatus sketched in figure 1 (*b*), with an insulated leading edge portion, was intended to permit the effects of cooling or heating the wall in the separated region to be studied without making the upstream profiles fuller or less full. It was thought that with this arrangement the pressure at separation, or at any rate that at the top of the laminar foot, would perhaps be reduced by cooling. However, as can be seen from figures 8 and 9, this was not found to be the case, probably because of the rapid adjustment in velocity profile that occurs at the temperature discontinuity, as described above.

One other point in connection with figures 5 to 9 deserves mention: the length of the laminar foot is virtually unaffected by wall temperature, and this means that the transition position is independent of heat transfer. It might perhaps have been expected that transition would be delayed by cooling. However, the 2·6 in. × 1·5 in.-tunnel in which the investigations were performed has a high turbulence level so that the stabilizing effect which cooling has on the laminar boundary layer oscillations, and the converse destabilizing effect of heating, may not much affect the position of transition. Alternatively it may be that since the length of the laminar foot is greater in terms of the upstream boundary layer thickness when the wall is cooled (the thickness then being reduced), cooling does in fact have a stabilizing effect with regard to transition.

(*b*) *Turbulent separations*

Figure 13 shows pressure distributions for turbulent separations. The pressure p at the wall, divided by p_1, the pressure in the undisturbed

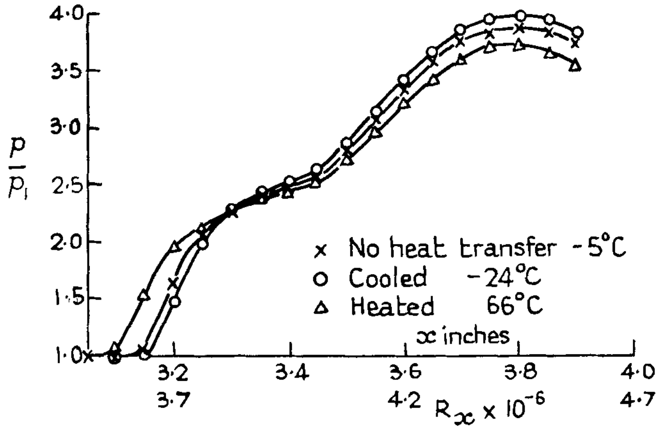

Figure 13. Pressure distributions with turbulent separation induced by a wedge in the mainstream. Stagnation pressure 5·7 atmospheres.

upstream boundary layer, is plotted against the distance x from the leading edge. The Reynolds numbers R_x based on x and free-stream conditions are marked on the abscissa scales. Separation was provoked by a shock wave generated by a short 15° wedge in the mainstream. Since the wedge was short, the shock was followed closely by an expansion, so the peak pressures in figure 13 are not as high as those corresponding to the regular reflection of a shock of 15° flow deflection angle. The pressure distributions have the characteristic shape for turbulent boundary layers with fairly large regions of separated flow (Gadd, Holder & Regan 1954). There is initial steep rise of pressure up to separation, downstream of which the pressure gradients fall off until the position where the shock strikes the boundary layer, and the pressure gradients then increase again until the peak pressure is reached.

Figure 14 shows Mach number traverses made at a distance of 3·8 in. from the leading edge in the undisturbed boundary layer, with the separation-provoking wedge removed. The Reynolds number R_x was $4{\cdot}4 \times 10^6$ at the measurement position. The Mach number was deduced from the static pressure at the wall and the readings of the pitot tube shown in figure 2. In figure 14 the Mach numbers M are plotted against the

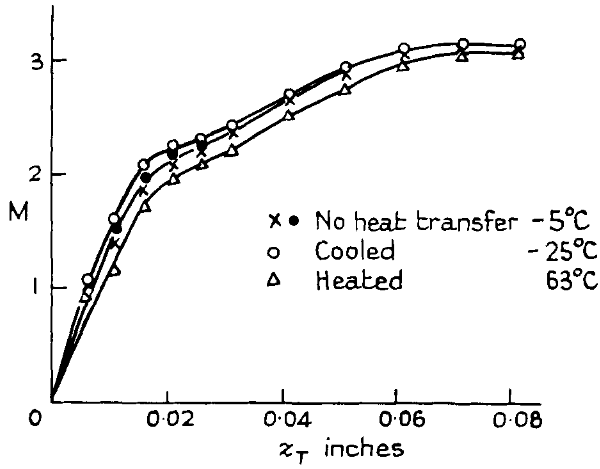

Figure 14. Turbulent Mach number profiles. Stagnation pressure 5·7 atmospheres.

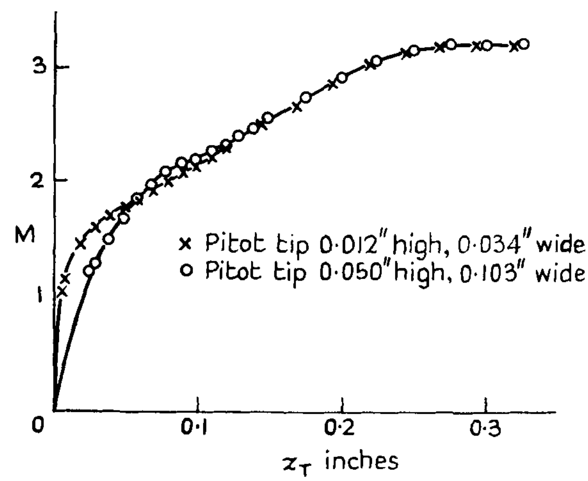

Figure 15. The effect of Pitot size on measured Mach number profiles in a turbulent tunnel-wall boundary-layer.

distances z_T from the wall to the centre of the Pitot tube opening The profiles are distorted near the wall due to pitot tube interference with the flow. This was established by traversing one of the thicker tunnel wall boundary layers with a small pitot tube and then with a large one whose dimensions and geometry relative to the tunnel wall boundary layer were roughly similar to those of the tube used for the flat plate traverses of figure 14 relative to the flat plate boundary layer. The tunnel wall results are shown in figure 15. The results obtained with the large tube show too small a gradient near the wall, and too sharp a 'shoulder' a little further out. There are evidently similar distortions in figure 14. Nevertheless the qualitative effects of heating or cooling are clear: heating the wall makes the boundary layer displacement thickness greater and reduces the velocities in the boundary layer. The slight reduction in free-stream Mach number in the heated cases is due to the static pressure at the wall (assumed to be constant across the boundary layer) being higher. This is probably caused by the increased rate of growth of the boundary layer and the consequent increased deflection of the free-stream.

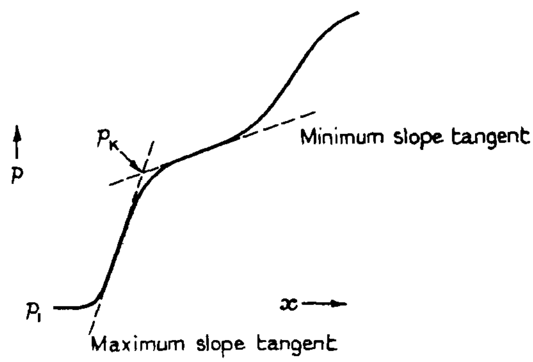

Figure 16. The definition of the kink pressure p_k from the pressure distibution with a considerable extent of separated turbulent flow.

The pressure distributions of figure 13 show that the upstream effect is greatest with the heated wall and smallest with the cooled wall. This is despite the fact that the maximum pressure at the downstream end of the distribution is somewhat reduced with the heated wall, and increased with the cooled wall. The variation in maximum pressure is probably associated with the effects of the expansion emanating from the downstream end of the shock-generating wedge. When the wall is heated the boundary layer displacement thickness is greater, and the expansion can produce effects further upstream, thus reducing the peak pressure. With a reduced peak pressure one would expect the shock to have a smaller upstream effect, other things being equal. However, the increased displacement thickness, besides enlarging the upstream region influenced by the expansion, also tends similarly to extend the influence of the shock further upstream, and this presumably more than counteracts the effect of the reduced peak pressure.

The effects on the pressure coefficient at the separation point may be gauged by considering the kink pressure p_k, defined as the intersection of the tangents of maximum and minimum slope, as in figure 16. It has been found in previous investigations (Gadd, Holder & Regan 1954) that p_k is close to the pressure at separation. From figure 13 it appears that the kink pressure ratio p_k/p_1 is approximately equal to 2·2 for the cooled and insulated cases, and to 2·1 for the heated case. This variation may be merely due to experimental scatter, though by analogy with the theoretical predictions for laminar layers it seems quite likely that there is some effect of heat transfer on the pressure coefficient for turbulent separation, such that heating reduces it and cooling increases it. However, much larger temperature differences than those used in the present experiments would be needed to produce very large effects.

4. Conclusions

The interesting result has been obtained that the pressure distribution at the wall with a laminar separation is approximately unaltered by wall temperature. It has to be remembered that the temperature range covered in the experiments was not very large, and in many practical cases there will be a much more drastic degree of cooling. Furthermore, the experiments were performed at one Mach number only ($M \doteqdot 3$), and with a high level of free-stream turbulence. The latter factor is important since the position of transition largely determines the upstream effect for flows which are laminar at separation but which turn turbulent before reattachment. In the absence, however, of obvious reasons for doubting the general applicability of the present findings, it may be tentatively assumed that they are always approximately valid for cases of laminar separation in supersonic flow with wall temperatures not too far from the zero heat transfer value, and with transition occurring in the separated layer. For such cases it would appear, therefore, that although as suggested by theory, there may well be a considerable effect of heat transfer on the pressure coefficient at separation, little sign of this appears in the overall pressure distribution. For practical purposes the pressure distribution is usually of greater interest than the precise position of separation, so the experimental result is very convenient.

For turbulent layers, the pressure coefficient at separation is probably decreased by heating and increased by cooling, though the effect is not large. The upstream effect produced by a given disturbance increases when the wall is heated and decreases when it is cooled. This appears to be associated with the parallel effect of wall temperature on boundary layer displacement thickness, an effect which is only likely to be large when the wall temperature differs very greatly from the zero heat transfer value.

Thus for estimating overall pressure distributions associated with boundary layer separation in supersonic flow when the wall is moderately heated or cooled, it is probably sufficiently accurate in most cases to use data obtained from experiments performed with zero heat transfer.

The work described above was carried out in the Aerodynamics Division of the National Physical Laboratory, and is published by permission of the Director of the Laboratory.

References

Bradfield, W. S., de Coursin, D. G. & Blumer, C. B. 1954 The effects of leading edge bluntness on a laminar supersonic boundary layer, *J. Aero. Sci.* **21**, 373.

Cohen, C. B. & Reshotko, E. 1955 a Similar solutions for the compressible laminar boundary layer with heat transfer and pressure gradient, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3325.

Cohen, C. B. & Reshotko, E. 1955 b The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3326.

Gadd, G. E. 1952 The numerical integration of the laminar compressible boundary layer equations, with special reference to the position of separation when the wall is cooled, *Aero. Res. Counc., Lond., Rep.* no. 15,101.

Gadd, G. E. 1956 a A theoretical investigation of the effects of Mach number, Reynolds number, wall temperature and surface curvature on laminar separation in supersonic flow, *Aero. Res. Counc., Lond., Rep.* no. 18,494.

Gadd, G. E. 1956 b A review of theoretical work relevant to the problem of heat transfer effects on laminar separation, *Aero. Res. Counc., Lond., Rep.* no. 18,495.

Gadd, G. E., Holder, D. W. & Regan J. D. 1954 An experimental investigation of the interaction between shock waves and boundary layers, *Proc. Roy. Soc.* A, **226**, 227.

Howarth, L. (Ed.). 1953 *Modern Developments in Fluid Dynamics. High Speed Flow.* Vol. 1. Oxford : Clarendon Press.

Illingworth, C. R. 1954 The effect of heat transfer on the separation of a compressible laminar boundary layer, *Quart. J. Mech. Appl. Math.* **7**, 8.

Morduchow, M. & Grape, R. G. 1955 Separation, stability, and other properties of compressible laminar boundary layer with pressure gradient and heat transfer, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3296.

Taylor, G. I. 1938 Measurements with a half Pitot tube, *Proc. Roy. Soc.* A, **166**, 476.